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# A Mixing Law to Model the Dielectrics Properties of Porous Media

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## ABSTRACT

A new mixing law has been developed to model the dielectric properties of saturated porous media. The total propagation time of an electromagnetic wave through the composite porous medium is expressed as the sum of the propagation times through each component (rock, water and oil). A power dependence is established between the propagation lengths and the volume fractions for each component, and is expressed in terms of a water saturation exponent $\beta_w$, an oil saturation exponent $\beta_n$, and a $\beta_g$ exponent corresponding to the rock matrix material. The mixing law predicts all the major features of the measured frequency ($\omega$) dependent dielectric constant as a function of water saturation. We conclude that the rapid increase of the dielectric constant at low water saturation values is consistent with $\beta_w \sim 0.3$, while a decrease at high water saturations is indicative of $\beta_n < 1$. The hysteresis observed in the dielectric constant during imbibition and drainage is associated with the existence of a different $\beta_n$ for each process. We further show that the exponent corresponding to the water fraction is related to the Archie exponents in the low frequency limit. Reported experimental data at two different frequencies were used to establish the values of the model parameters at those frequencies.

## INTRODUCTION

Several mixing laws have been proposed to estimate rock properties such as porosity and water saturation from electrical measurements in the lab and at the wellbore.[1] The best known of these, Archie's Law,[2] relies on exponents that depend strongly on the geometry of the pore space, while other mixing laws such as CRIM [3] (Complex Refractive Index Method) do not have geometry-dependent parameters. Archie's law has been used extensively since it was introduced in 1942, and from the vast quantity of experimental data and models, various hypotheses have been proposed to relate the cementation and saturation exponents

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References and illustrations at end of paper.

with pore space geometry.[4,5] The amount of published data to back up CRIM is not so abundant, and those in the medium frequency range (10 KHz to 1 MHz) show considerable disagreement with the model.[6,7] In this paper we propose a modified form of CRIM, introducing parameters that relate the distance traveled by the electromagnetic wave in each component with the corresponding volume fraction. By choosing appropriate values for these exponents, we can reproduce reported dielectric constant measurements.[6]

## COMPLEX REFRACTIVE INDEX METHOD

One of the most commonly used mixing laws in the interpretation of dielectric well logs is CRIM, defined by the equation

$$\sqrt{\varepsilon^*} = S_w\phi\sqrt{\varepsilon_w^*} + (1 - S_w)\phi\sqrt{\varepsilon_n} + (1 - \phi)\sqrt{\varepsilon_g} \quad (1)$$

where $\varepsilon^*$ is the complex dielectric constant of the mixture, $\phi$ is the porosity, $S_w$ is the water saturation and $\varepsilon_w^*$, $\varepsilon_n$ and $\varepsilon_g$ are the dielectric constants of water, oil and rock. To derive this equation, we assume that the electromagnetic wave travels distances $l_w$, $l_n$ and $l_g$ through a three-component parallel layered medium. The total travel time through the three components is $t = t_w + t_n + t_g$. Expressing $t = \frac{l}{v}$ in terms of travel distance $l$, and considering that $v = \frac{c}{n^*}$, where $v$ is the wave velocity in a medium of complex refractive index $n^*$ and $c$ is the velocity of light in vacuum, we obtain

$$n^* = \frac{l_w}{l}n_w^* + \frac{l_n}{l}n_n + \frac{l_g}{l}n_g. \quad (2)$$

Substituting $n^* = \sqrt{\varepsilon^*}$ in Equation 2 we obtain

$$\sqrt{\varepsilon^*} = \frac{l_w}{l}\sqrt{\varepsilon_w^*} + \frac{l_n}{l}\sqrt{\varepsilon_n} + \frac{l_g}{l}\sqrt{\varepsilon_g}. \quad (3)$$

Notice that Equation 3 relates the dielectric constant of the medium with the dielectric constants of each component through parameters that depend on the spatial arrangement of the components. The task now is to find a relationship between these coefficients and the geometry. For the simple case in which incidence is at right angle on a set of three parallel layers of water, oil and rock, this relationship is trivial and can be written as the volume fractions

$$\begin{aligned}\frac{l_w}{l} &= S_w\phi \\ \frac{l_n}{l} &= (1 - S_w)\,\phi \\ \frac{l_g}{l} &= 1 - \phi\end{aligned} \quad (4)$$

for each component. This leads to the equation that defines CRIM (Equation 1). If the incidence is not at right angle on the parallel layered medium, Equations 4 are not applicable because the lengths will now depend on the incidence angle and the refractive index of the layers. This extreme case illustrates the limitation of CRIM in not being able to consider the pore geometry. In the pore space, the mixture of grains and fluids constitute a relatively homogeneous material. Therefore, even if the distances are not the same as the volume fractions, the difference may be small enough to be able to use CRIM as a good approximation in some cases. However, there are too few published experimental results to be able to say anything conclusive. Dielectric measurements in the 60 KHz to 4 MHz range are not consistent with CRIM,[6,7] while higher frequency measurements show some agreement.[8] Our theoretical modeling indicates that CRIM is a good approximation in the static (low frequency) case,[9] and also at high frequencies,[10] but not at the intermediate frequencies (60 KHz to 4 MHz.).

## A NEW MIXING LAW

In the previous section we saw the need for a more general form of the travel distance ratios in Equation 2 in terms of the volume fractions (Equation 4) and considering the pore space geometry. To do this, we now propose the following general relations

$$\begin{aligned} \frac{l_w}{l} &= \sum_i b_{wi} \left(S_w \phi\right)^{\beta_{wi}} \\ \frac{l_n}{l} &= \sum_i b_{ni} \left[\left(1 - S_w\right) \phi\right]^{\beta_{ni}} \\ \frac{l_g}{l} &= \sum_i b_{gi} \left(1 - \phi\right)^{\beta_{gi}} \end{aligned} \tag{5}$$

where we introduce the b coefficients and the $\beta$ exponents which contain information about the geometry for each component of the medium. Substituting Equations 5 into Equations 3 we obtain

$$\begin{aligned} \sqrt{\varepsilon^*} = &\sum_i b_{wi} \left(S_w \phi\right)^{\beta_{wi}} \sqrt{\varepsilon_w^*} \\ &+ \sum_i b_{ni} \left[\left(1 - S_w\right) \phi\right]^{\beta_{ni}} \sqrt{\varepsilon_n} \\ &+ \sum_i b_{gi} \left(1 - \phi\right)^{\beta_{gi}} \sqrt{\varepsilon_g}. \end{aligned} \tag{6}$$

We also propose that for a given pore space geometry there exists one dominating term in each summation. For example, CRIM is obtained from Equation 6 by choosing all coefficients b and all exponents $\beta$ equal to one. In the dc limit ($\omega \rightarrow 0$), Equation 6 leads to the expression for the real conductivity $\sigma$

$$\sigma = \left[\sum_i b_{wi} \left(\phi S_w\right)^{\beta_{wi}}\right]^2 \sigma_w, \tag{7}$$

where $\sigma_w$ is the dc conductivity of water. If we compare Equation 7 with Archie's equation

$$\sigma = \left[a S_w^n \phi^m\right] \sigma_w, \tag{8}$$

we notice that they are equivalent if the series in Equation 7 has only one term, and if $a = b_w^2$ and $m = n = 2\beta_w$.

The two previous examples, CRIM and Archie's law, derive directly from our model if we assume that there is only one dominant term in each series of Equation 6, for which the mixing law takes the form

$$\begin{aligned} \sqrt{\varepsilon^*} = &\, b_w \left(S_w \phi\right)^{\beta_w} \sqrt{\varepsilon_w^*} \\ &+ b_n \left[\left(1 - S_w\right) \phi\right]^{\beta_n} \sqrt{\varepsilon_n} \\ &+ b_g \left(1 - \phi\right)^{\beta_g} \sqrt{\varepsilon_g}. \end{aligned} \tag{9}$$

The dielectric constants of oil and rock are taken as real quantities because of the low values of their conductivities. Applications of this simplified form of the mixing law are discussed in the following section.

## ANALYSIS OF SATURATION DATA

Figure 1 shows typical dielectric constant data from measurements at 105 KHz,[6] and Figure 2 shows the corresponding curve calculated with the CRIM method. It is obvious from the comparison of these two curves that the magnitudes of the experimental data are much greater than the predicted CRIM values, except at very low water saturations, and that the curves have opposite curvatures. CRIM does not seem to be a good model at this frequency (although there seems to be some agreement at 1.1 GHz).[8]

The most significant term in Equation 9 corresponds to the water contribution (except for very low water saturations), and therefore the most important parameters are $\beta_w$ and $b_w$. Figure 3 shows the dependence of the dielectric constant with water saturation calculated with Equation 9 for several values of $\beta_w$ and keeping all other values of b and $\beta$ equal to 1. The curvature and the value of the dielectric constant change with $\beta_w$, approximating the general form of the experimental line shape for $\beta \sim 0.3$. The lowest curve in Figure 3 is equivalent to CRIM ($\beta_w = 1$). In Figure 4 we show the result of varying $b_w$ for $\beta_w = 0.3$. The major effect is a change in amplitude and a variation of the slope at high water saturations. From the previous observations we conclude that it must be possible to fit the experimental data with a suitable choice of the $\beta$ and b parameters. Figure 5 shows a fit to the data in

Figure 1 with $\beta_w = 0.23$ and $b_w = 0.382$. The fit is excellent for high water saturations, and follows the steep increase at low $S_w$ values.

Another important feature of the experimental data[11] is the decrease of the dielectric constant at high values of $S_w$ and the hysteresis in the imbibition and drainage processes (Figure 6). These effects can be explained introducing different values of $\beta_n$ for each of these two processes. Figure 7 shows the calculated dielectric constant for $\beta_w = 0.3$ and two different values of $\beta_n$ (0.3 and 1.0), following the qualitative form of the experimental results in Figure 6. It is difficult to speculate about the geometrical distributions of water and oil (or air) giving rise to a decrease in $\varepsilon$ at high water saturation, and the hysteresis in the drainage process. More experimental work is clearly needed.

The fit to two dielectric constant curves measured on the same sample a two different frequencies[6] is shown in Figure 8. The values of the parameters are $b=0.44$, $\beta=0.1$ for the data at 105 KHz, and $b=0.336$, $\beta=0.23$ for the data at 1.2 MHz.

## CONCLUSIONS

We have shown that, with the proposed model, it is possible to fit the major features of experimental dielectric constant vs. water saturation results by adjusting the model b and $\beta$ parameters. The mixing law allows for the particular pore geometry of a formation to be taken into consideration when interpreting electromagnetic log data. This can done by analyzing a small length of core to evaluate the model parameters for a particular formation. Further research is needed to relate the model parameters with pore space geometry.

## NOMENCLATURE

$b_w$, $b_n$, $b_g$ = Model linear coefficients.

$\beta_w$, $\beta_w$, $\beta_w$ = Model exponents.

$\sigma$ = Conductivity.

$\varepsilon$ = Dielectric constant.

$S_w$ = Water saturation.

$\phi$ = Porosity.

$l$, $l_w$, $l_n$, $l_g$ = Travel distances.

$t$, $t_w$, $t_n$, $t_g$ = Travel times.

$v$ = Wave velocity.

c = Speed of light.

n = Refractive index.

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## REFERENCES

1. Banhegyi, G.: "Comparison of electrical mixture rules formulae", Colloid & Polymer Science (1988) volume 266, 11.

2. Archie, G.E.: "The electrical resistivity log as an aid in determining some reservoir characteristics", Transactions of the AIME (1942) volume 146, 54.

3. Wharton, R.P., Hazen, G.A., Rau, R.N., Best, D.L.: "Electromagnetic propagation Logging: advances in technique and interpretation", Soc. Pet. Eng. of AIME (1980) paper 9267.

4. Sen, P.N., Scala, C., Cohen, M.H.: "A self-similar model for sedimentary rocks with application to the dielectric constant of fused glass beads", Geophysics (1981) volume 46, 781.

5. Roberts, J.N., Schwartz, L.M.: "Grain consolidation and electrical conductivity in porous media", Phys. Rev. B (1985) volume 31, 5990.

6. Knight, R., Endres, A.: "The effect of rock/water interaction in modeling the dielectric response of sandstones", SPWLA Thirtieth Annual Logging Symposium (1989) Paper JJ.

7. Knight, R.J., Nur, A.: "The dielectric constant of sandstones, 60 KHz to 4 MHz", Geophysics (1987) volume 52, 644.

8. Rau, R.N., Wharton, R.P.: "Measurement of core electrical parameters at UHF and microwave frequencies", Soc. Pet. Eng. of AIME (1980) paper 9380.

9. Vega, S., Rodriguez, A., Abreu, R.A.: "Numerical simulation of the dielectric behavior of slightly heterogeneous media", Intevep (1990) technical report.

10. Rodriguez, A.: "Numerical simulations of the complex dielectric constant of heterogeneous media", Intevep (1990) technical report.

11. Knight, R., Nur,A.: "Geometrical effects in the dielectric response of partially saturated sandstones", The Log Analyst (1987) volume 28, 513.

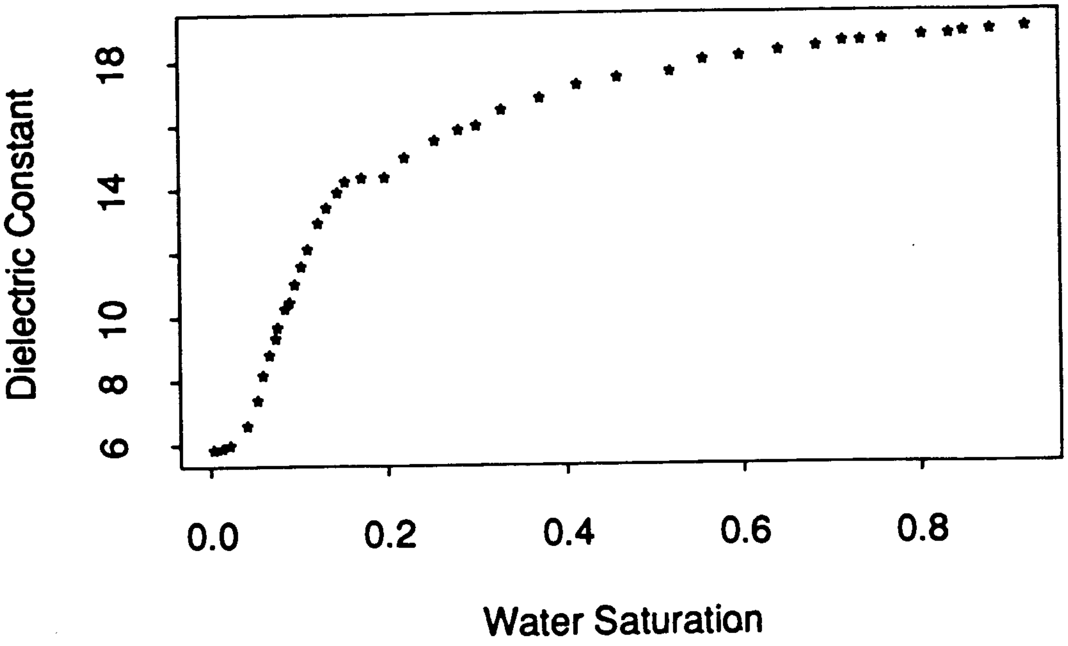

Figure 1. Dielectric constant measured at 105 KHz, taken from Reference 6.

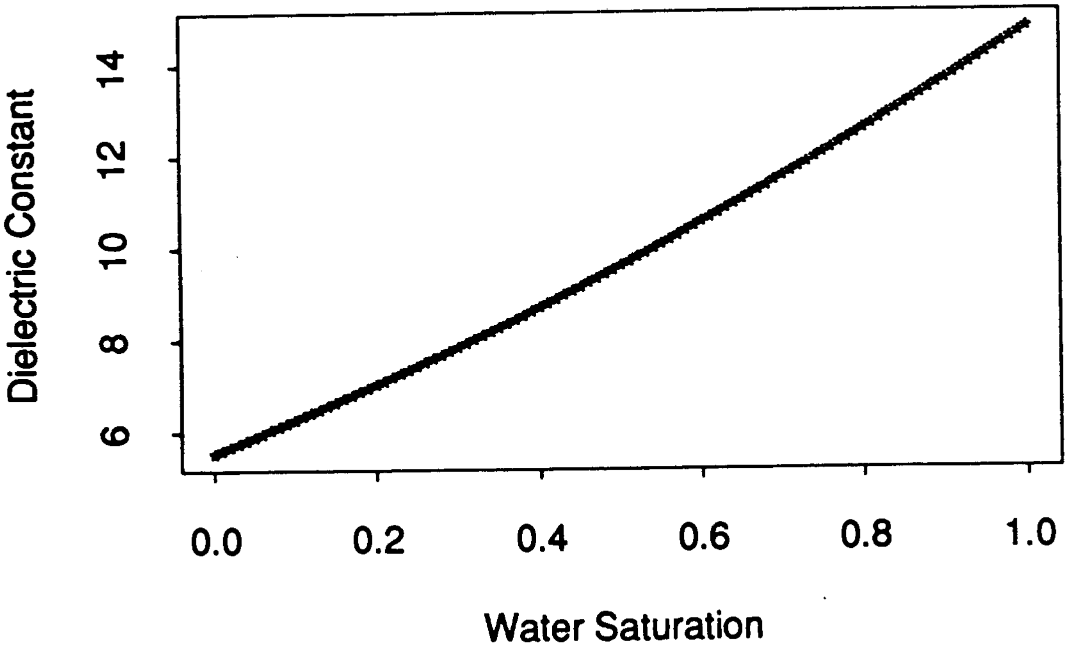

Figure 2. Dielectric constant as a function of $S_w$ calculated with the CRIM method.

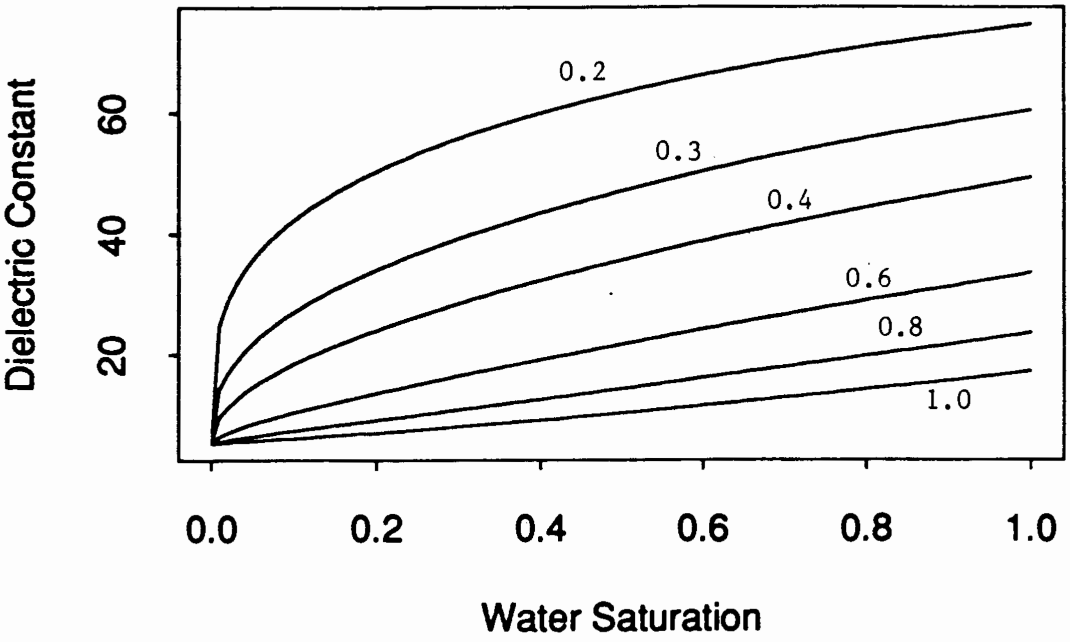

Figure 3. Dielectric constant as a function of $S_w$ calculated for diferent values of $\beta_w$ keeping all other b and $\beta$ parameters equal to one.

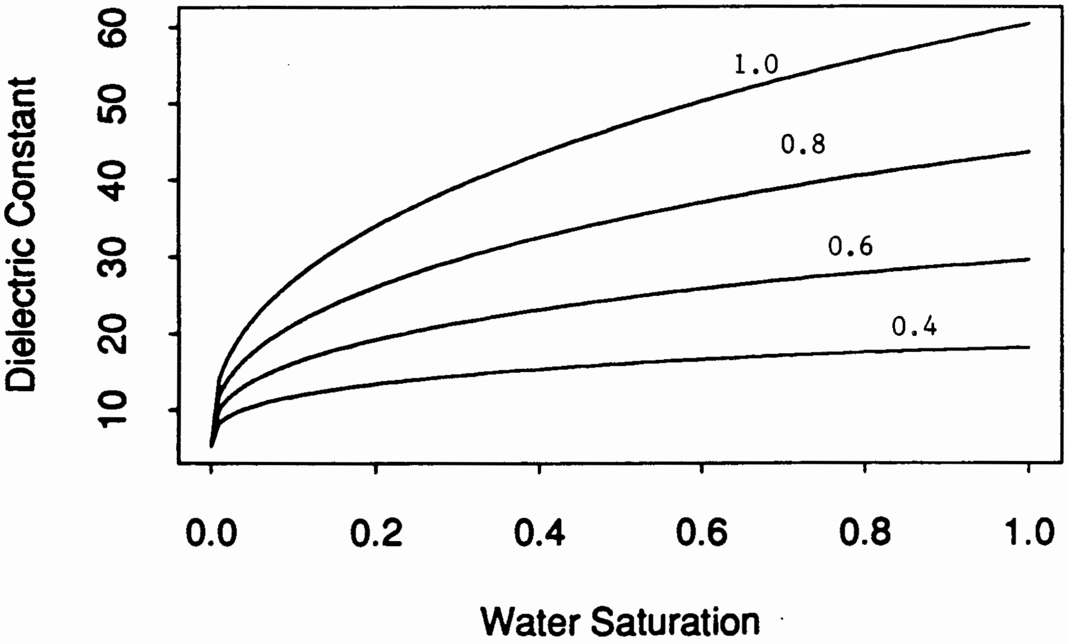

Figure 4. Variation of the dielectric constant as a function of $S_w$ for different values of $b_w$ for $\beta_w$ = 0.3.

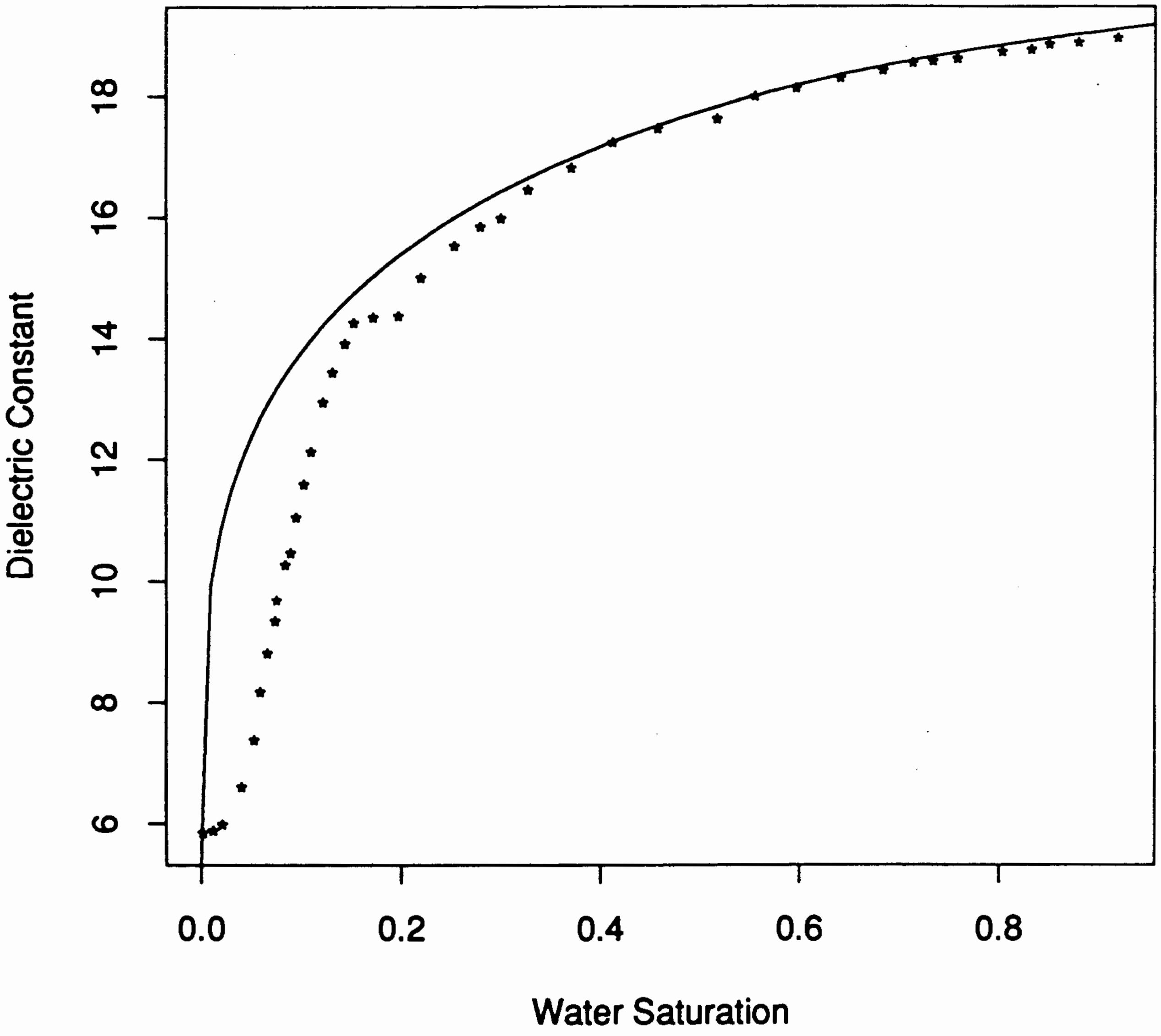

Figure 5. Fit to the data in Figure 1 with $\beta_w = 0.23$ and $b_w = 0.382$.

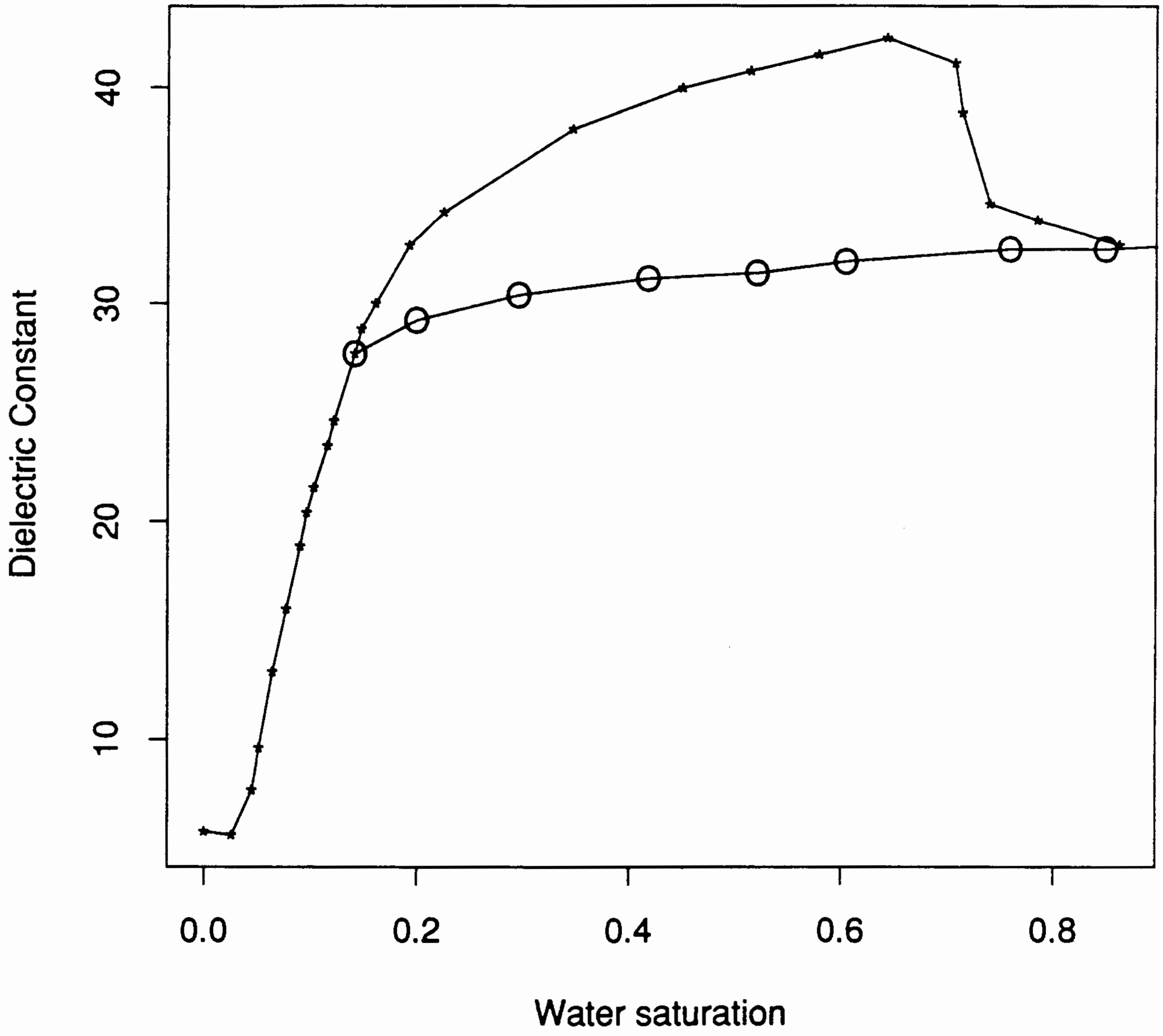

Figure 6. Hysteresis in the dielectric constant, taken from Reference 6.

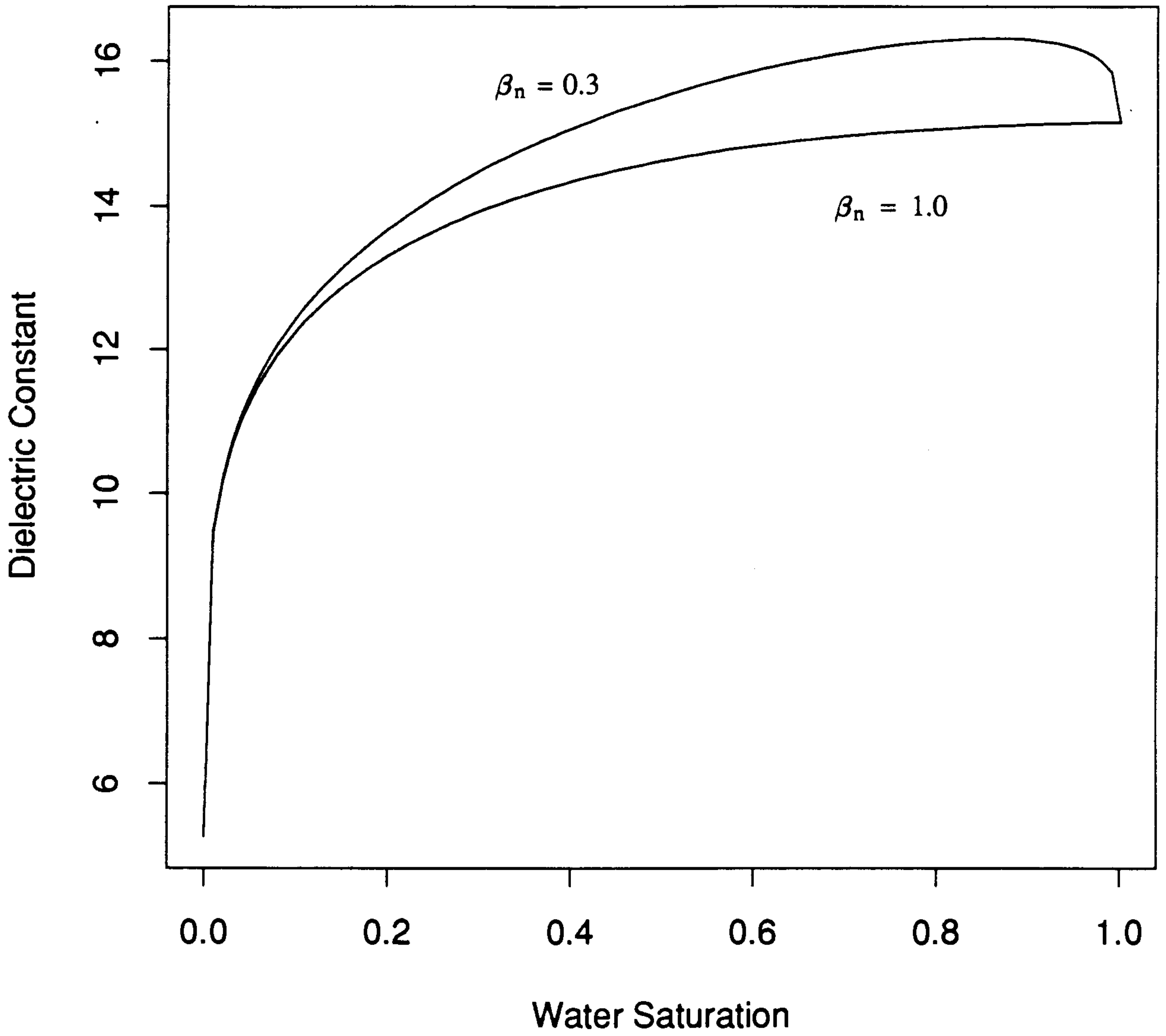

Figure 7. Calculated dielectric constant for $\beta_w = 0.3$ and two values of $\beta_n$ (0.3 and 1.0).

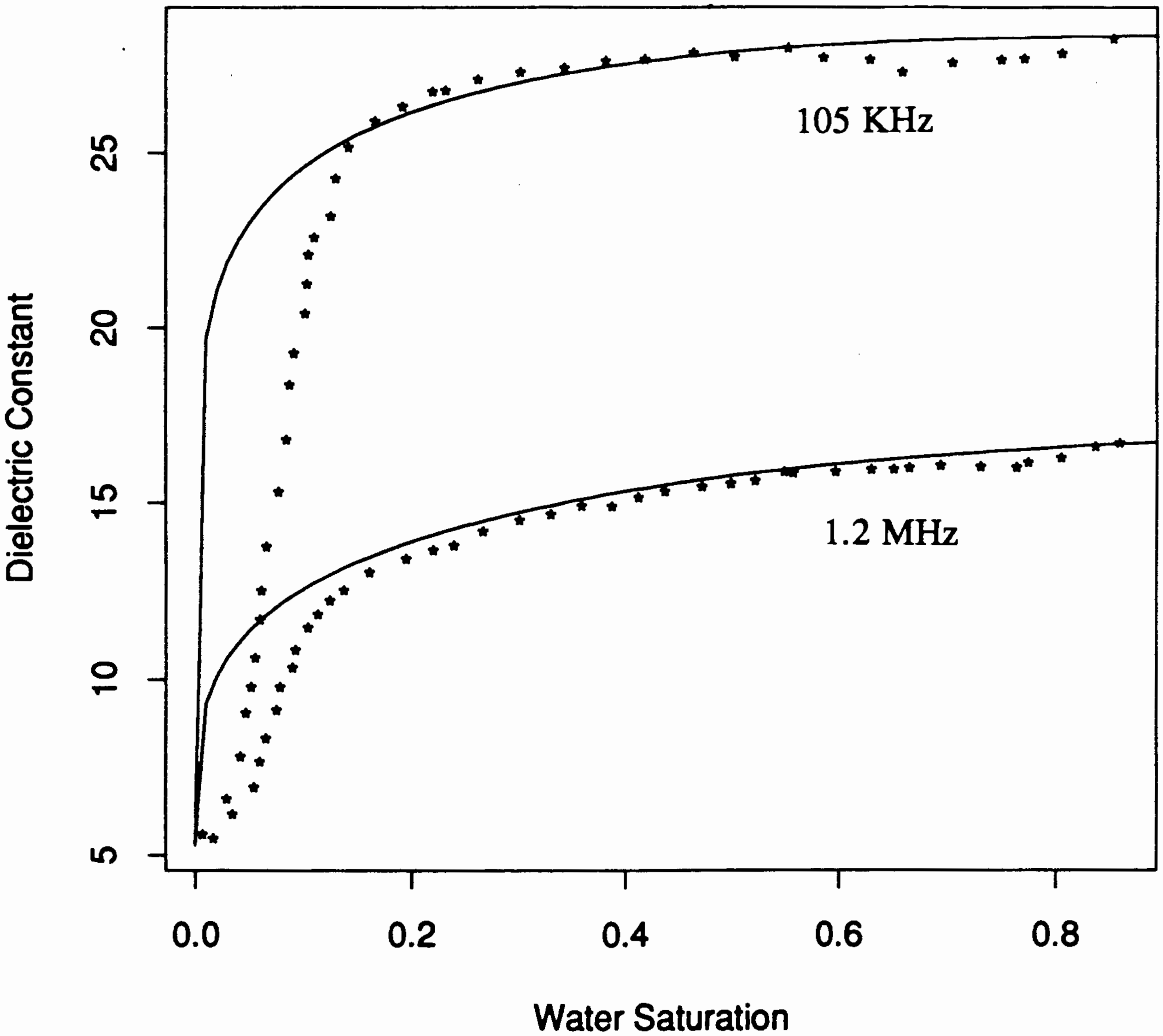

Figure 8. Fit to two dielectric constant curves (Reference 6) measured at different frequencies.